STOCKHOLM UNIVERSITY
Department of Statistics
Ellinor Fackle-Fornius

EXAM IN MULTIVARIATE METHODS
November 5 2019

Time: 5 hours

Aids allowed: Pocket calculator, language dictionary.

The exam consists of five questions. To score maximum points on a question solutions need to be clear, detailed and well motivated.

Results will be announced no later than November 19.

Question 1. (16 points)

Given the following data, compute the euclidean, statistical and Mahalanobis distance between observations 2 and 4 and observations 2 and 3. Which set of observations is more similar? Why?

Obs.	X_1	X_2
1	7	8
2	3	1
3	9	8
4	2	4
5	5	5

Question 2. (16 points)

Data were collected on x_1 =sales (billions) and x_2 =profits (billions) for the ten largest companies in the world. The sample mean vector and covariance matrix are given by

$$\bar{\mathbf{x}} = \begin{pmatrix} 155.60 \\ 14.70 \end{pmatrix} \quad \mathbf{S} = \begin{pmatrix} 7476.45 & 303.62 \\ 303.62 & 26.19 \end{pmatrix}.$$

The eigenvalues of the covariance matrix are

$$\lambda_1 = 7488.8$$
$$\lambda_2 = 13.8$$

a) Compute the correlation matrix $\mathbf{R}$ and the eigenvalues of $\mathbf{R}$.

b) Find the proportion of the total variance explained by the first PC based on the correlation matrix.

c) Find the proportion of the total variance explained by the first PC based on the covariance matrix.

d) Discuss when it is appropriate to base the PCA on the covariance/correlation matrix.

Question 3. (16 points)

Forty engineers were given six tests and measurements were taken on the following six variables

x_1 = intelligence $\quad x_4$ = dotting
x_2 = form relations $\quad x_5$ = sensory motor coordination
x_3 = dynamometer $\quad x_6$ = perseveration.

An exploratory factor analysis was performed. The analysis resulted in a two factor solution with the rotated (rounded) pattern loadings presented in the table below. The two factors are assumed to be orthogonal.

	F_1	F_2
x_1	0.14	0.69
x_2	-0.64	0.61
x_3	0.56	0.12
x_4	0.80	0.16
x_5	-0.07	-0.58
x_6	0.49	0.54

a) What are the usual assumptions for the factor model?

b) Compute the unique variances.

c) Compute the estimated/reproduced correlation matrix.

d) If another factor rotation was performed, would you expect any of your results in b) or c) to change?

Question 4. (16 points)

For the data set in Question 1 the following matrix of distances can be calculated

$$
\mathbf{D} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix}
0 & & & & \\
8.0623 & 0 & & & \\
2.0000 & 9.2195 & 0 & & \\
6.4031 & 3.1623 & 8.0623 & 0 & \\
3.6056 & 4.4721 & 5.0000 & 3.1623 & 0
\end{pmatrix}
\end{array}
$$

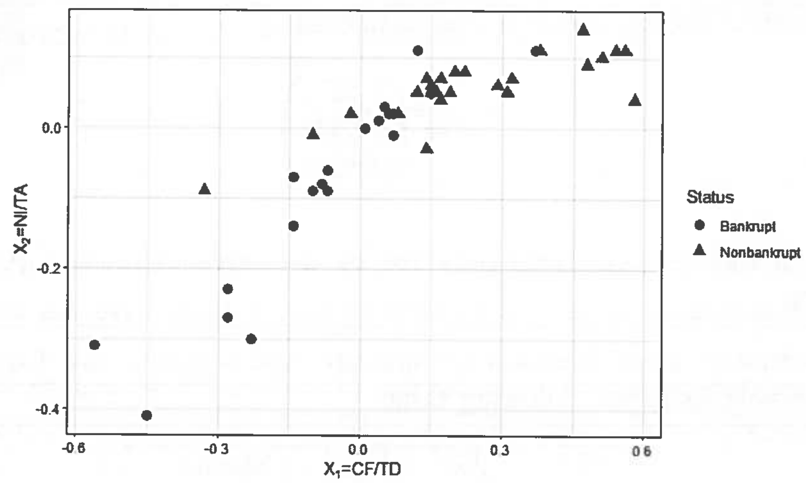
a) Cluster the 5 observations using the Complete linkage method.

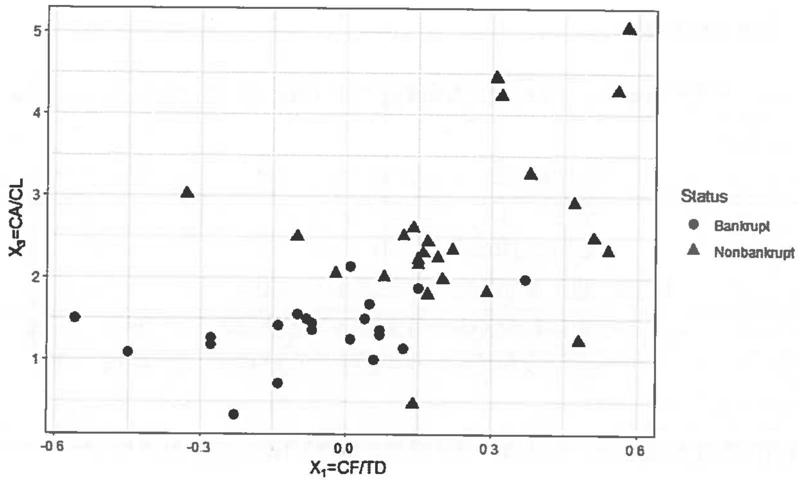
b) Draw the dendrogram of the clustering in a).

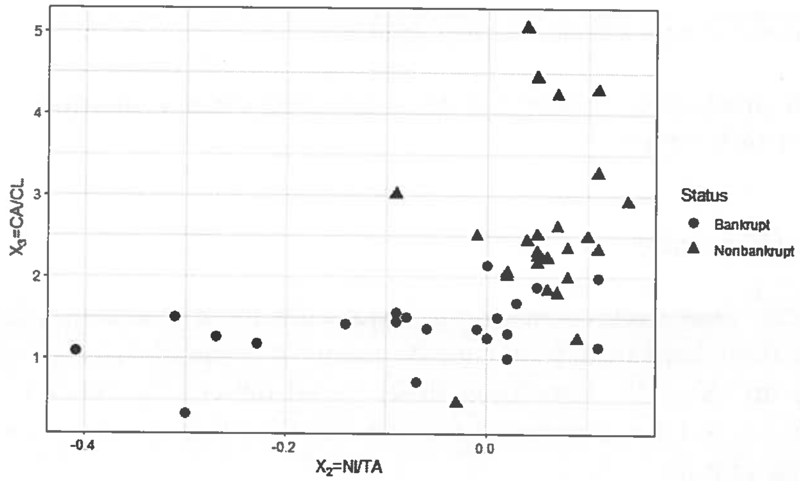
c) Can cluster analysis be considered as a data reduction technique? Compare with other data reduction techniques.

Question 5. (16 points)

Annual financial data were collected on 3 variables for 21 bankrupt firms approximately 2 years prior to their bankruptcy and for 25 financially sound firms at about the same time. The variables are X_1=CF/TD=(cash flow)/(total debt), X_2=NI/TA=(net income)/(total assets), and X_3=CA/CL=(current assets)/(current liabilities). The data are plotted pairwise in the following graphs.







Fisher's linear discriminant function was estimated to

$$\gamma = \begin{pmatrix} 1.1278 \\ 3.6946 \\ 0.8265 \end{pmatrix}.$$

a) Comment on the discrimination provided by the three variables individually as well as jointly in pairs.

b) Use the estimated linear discriminant function and compute the discriminant scores for the 6 new observations in the following table.

X_1	X_2	X_2	Status
-0.45	-0.41	1.09	Bankrupt
0.51	0.10	2.49	Nonbankrupt
-0.56	-0.31	1.51	Bankrupt
0.08	0.02	2.01	Nonbankrupt
0.06	0.02	1.01	Bankrupt
0.38	0.11	3.27	Nonbankrupt

c) The means of the discriminant scores in the original data set was 0.7509 for the bankrupt firms and 2.6143 for the nonbankrupt firms. Assume that the prior probabilities are proportional to the sample proportions and that the cost to misclassify a bankrupt firm as nonbankrupt is 3 while the cost to misclassify a nonbankrupt firm as bankrupt is 1. Classify the 6 new observations in the table.

d) Determine the confusion matrix and accuracy of the classification in c).

Formula Sheet for the Exam in Multivariate Methods

Vectors and matrices

- Length of a vector $\mathbf{a} = (a_1, a_2, \ldots, a_p)$

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_p^2}$$

- Determinant of a 2×2 matrix $\mathbf{A}$

$$\det(\mathbf{A}) = |\mathbf{A}| = a_{11}a_{22} - a_{12}a_{21}$$

- Inverse of a 2×2 matrix $\mathbf{A}$

$$\mathbf{A}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

- Eigenvalues are the roots of the characteristic equation

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \mathbf{0}$$

  For each eigenvalue the solution to

$$(\mathbf{A} - \lambda\mathbf{I})\,\mathbf{x} = \mathbf{0}$$

  gives the associated eigenvector $\mathbf{x}$

Distances

- Euclidean

$$D_{ik} = \sqrt{\sum_{j=1}^{p} (x_{ij} - x_{kj})^2}$$

- Statistical

$$SD_{ik} = \sqrt{\sum_{j=1}^{p} \left(\frac{x_{ij} - x_{kj}}{s_j} \right)^2}$$

- Mahalanobis

$$MD_{ik} = \sqrt{(\mathbf{x}_i - \mathbf{x}_k)^T \mathbf{S}^{-1} (\mathbf{x}_i - \mathbf{x}_k)}$$

For $p = 2$

$$MD_{ik} = \sqrt{\frac{1}{1-r^2} \left[\frac{(x_{i1} - x_{k1})^2}{s_1^2} + \frac{(x_{i2} - x_{k2})^2}{s_2^2} - \frac{2r\,(x_{i1} - x_{k1})\,(x_{i2} - x_{k2})}{s_1 s_2} \right]}$$

Mean-correction and covariance

- Mean-corrected data

$$\underset{(n \times p)}{\mathbf{X}_m} = \{x_{ij}\} = \{X_{ij} - \bar{X}_j\}$$

- Covariance

$$\underset{(p \times p)}{\mathbf{S}} = \{s_{ij}\} = \left\{ \frac{\sum_{i=1}^{n} x_{ij} x_{ik}}{n-1} \right\} = \frac{\mathbf{SSCP}}{df} = \frac{1}{n-1} \mathbf{X}_m^T \mathbf{X}_m$$

Group Analysis

- Total sum of squares and cross products

$$\mathbf{SSCP}_{\text{total}} = \mathbf{SSCP}_{\text{within}} + \mathbf{SSCP}_{\text{between}}$$

- Pooled within-group sum of squares and cross products

$$\mathbf{SSCP}_{\text{within}} = \sum_{\ell=1}^{g} \mathbf{SSCP}_{\ell}$$

- Pooled covariance matrix

$$\mathbf{S}_{\text{pooled}} = \frac{\mathbf{SSCP}_{\text{within}}}{n-g}$$

- Between-group sum of squares and cross products

$$\mathbf{SSCP}_{\text{between}} = \mathbf{SSCP}_{\text{total}} - \mathbf{SSCP}_{\text{within}}$$

For $g = 2$ groups

$$\mathbf{SSCP}_{\text{between}} = \frac{n_1 n_2}{n_1 + n_2} (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)^T$$

Factor Analysis

- For the two-factor model

$$Var(x) = \lambda_1^2 + \lambda_2^2 + Var(\epsilon) + 2\lambda_1\lambda_2\phi$$

$$Cor(x, \xi_1) = \lambda_1 + \lambda_2\phi$$

$$Cor(x_j, x_k) = \lambda_{j1}\lambda_{k1} + \lambda_{j2}\lambda_{k2} + (\lambda_{j1}\lambda_{k2} + \lambda_{j2}\lambda_{k1})\phi$$

- RMSR for EFA

$$RMSR = \sqrt{\frac{\sum_{i=1}^{p} \sum_{j=i+1}^{p} res_{ij}^2}{p(p-1)/2}}$$

- RMSR for CFA

$$RMSR = \sqrt{\frac{\sum_{i=1}^{p} \sum_{j=i}^{p} (s_{ij} - \hat{\sigma}_{ij})^2}{p(p+1)/2}}$$

Two-Group Discriminant Analysis

- Maximize

$$\lambda = \frac{\gamma^T \mathbf{B} \gamma}{\gamma^T \mathbf{W} \gamma}$$

- Fisher's linear discriminant function

$$\gamma^T = (\mu_1 - \mu_2)^T \mathbf{\Sigma}^{-1}$$

- Wilks' Λ

$$\Lambda = \frac{|\mathbf{SSCP}_w|}{|\mathbf{SSCP}_t|}$$

$$F = \left(\frac{1-\Lambda}{\Lambda}\right)\left(\frac{n_1 + n_2 - p - 1}{p}\right) \sim F(p, n_1 + n_2 - p - 1)$$

- Classification based on decision theory: assign the observation to group 1 if

$$Z \geq \frac{\bar{Z}_1 + \bar{Z}_2}{2} + \ln\left[\frac{p_2 C(1|2)}{p_1 C(2|1)}\right]$$

Logistic regression

- Odds of the event $Y = 1$

$$odds = \frac{p}{1-p}$$

where

$$p = P(Y = 1)$$

- Probability of the event $Y = 1$ as a function of the explanatory variables

$$P(Y = 1|X_1, X_2, \ldots, X_k) = \frac{1}{1 + e^{-(\beta_0+\beta_1 X_1+\cdots+\beta_k X_k)}}$$

Quadratic equation

- The roots of the quadratic equation $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Department of Statistics

Stockholms universitet

Correction sheet

Date: 5/11 - 2019

Room: Uggleviksalen

Exam: Multivariate Methods (re-exam)

Course: Multivariate Methods

Anonymous code: 0008-AdX

☒ I authorise the anonymous posting of my exam, in whole or in part, on the department homepage as a sample student answer.

NOTE! ALSO WRITE ON THE BACK OF THE ANSWER SHEET

Mark answered questions

1	2	3	4	5	6	7	8	9	Total number of pages
X	X	X	X	X					10
Teacher's notes 14	15	13	15	14					

Points	Grade	Teacher's sign.
71		

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 1

Euclidean distance between 2 and 4: (Question 1)

$$\sqrt{(3-2)^2+(1-4)^2} = \sqrt{10}$$

R

Euclidean distance between 2 and 3:

$$\sqrt{(3-9)^2+(1-8)^2} = \sqrt{85}$$

R

$$\bar{X}_1 = \frac{7+3+9+2+5}{5} = 5{,}2$$

R

$$\bar{X}_2 = \frac{8+1+8+4+5}{5} = 5{,}2$$

$$S_1 = \sqrt{\frac{(7-5{,}2)^2+(3-5{,}2)^2+(9-5{,}2)^2+(2-5{,}2)^2+(5-5{,}2)^2}{5-1}} = \sqrt{8{,}2}$$

R

$$S_2 = \sqrt{\frac{(8-5{,}2)^2+(1-5{,}2)^2+(8-5{,}2)^2+(4-5{,}2)^2+(5-5{,}2)^2}{5-1}} = \sqrt{8{,}7}$$

The statistical distance between 2 and 4:

$$\sqrt{\left(\frac{3-2}{\sqrt{8{,}2}}\right)^2+\left(\frac{1-4}{\sqrt{8{,}7}}\right)^2} \approx 1{,}075$$

R

The statistical distance between 2 and 3:

$$\sqrt{\left(\frac{3-9}{\sqrt{8{,}2}}\right)^2+\left(\frac{1-8}{\sqrt{8{,}7}}\right)^2} \approx 3{,}166$$

R

$$S_{12} = \frac{(7-5{,}2)(8-5{,}2)+(3-5{,}2)(1-5{,}2)+(9-5{,}2)(8-5{,}2)+(2-5{,}2)(4-5{,}\ )+(-0{,}2)(-4{,}8)}{5-1}$$

$$= \frac{28{,}8}{5-1} = 7{,}2$$

R

$$r = \frac{7{,}2}{\sqrt{8{,}2 \cdot 8{,}7}} \approx 0{,}85$$

R

$\longrightarrow$

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 2

1) The Mahalanobis distance between 2 and 4:

$$MD_{24} = \sqrt{\frac{1}{1-0{,}85^2} \cdot \left[\frac{(3-2)^2}{8{,}2} + \frac{(1-4)^2}{8{,}7} - \frac{2 \cdot 0{,}85(3-2)(1-4)}{\sqrt{8{,}2} \cdot \sqrt{8{,}7}} \right]}$$

$\approx 2{,}519$

R

The Mahalanobis distance between 2 and 3:

$$MD_{23} = \sqrt{\frac{1}{1-0{,}85^2} \cdot \left[\frac{(3-9)^2}{8{,}2} + \frac{(1-8)^2}{8{,}7} - \frac{2 \cdot 0{,}85(3-9)(1-8)}{\sqrt{8{,}7} \cdot \sqrt{8{,}2}} \right]}$$

$\approx 2{,}378$

R

The variation in X_1 is smaller than X_2, thus X_1 has a set of observations that are more similar regarding variation. But not by much since there is not that large difference between the variances.

14

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AAX Sheet number: 3

2)

a) The diagonal terms in the correlation matrix will become 1, since

$$\frac{7476,45}{\sqrt{7476,45 \cdot 7476,45}} = 1 \qquad \frac{26,19}{\sqrt{26,19 \cdot 26,19}} = 1$$

R

The off-diagonal terms will be

$$\frac{303,62}{\sqrt{7476,45 \cdot 26,19}} \approx 0,686$$

R

$$R = \begin{pmatrix} 1 & 0,686 \\ 0,686 & 1 \end{pmatrix}$$

• We want to solve $\det(R-\lambda I)=0$ to obtain the eigenvalues.

R

$$\det(R-\lambda I) = \det\left(\begin{pmatrix} 1 & 0,686 \\ 0,686 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} 1-\lambda & 0,686 \\ 0,686 & 1-\lambda \end{pmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 0,686 \\ 0,686 & 1-\lambda \end{vmatrix} = (1-\lambda)(1-\lambda) - 0,686 \cdot 0,686$$

$$= 1 - 2\lambda + \lambda^2 - 0,470596$$

$$\lambda^2 - 2\lambda + 0,529404 = 0$$

R

$$\lambda_{1,2} = \frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 0,529404}$$

$$= 1 \pm 0,686$$

$\lambda_1 = 1,686$

$\lambda_2 = 0,314$

R

8

SU, DEPARTMENT OF STATISTICS

Room: UG7 Anonymous code: 0008-A0X Sheet number: 4

2b) The proportion explained by the first PC based on R is

$$\frac{1,686}{\underbrace{2}_{=p}} = 0,843$$

R

2

c) The proportion explained by the first PC based on cov. matrix is

$$\frac{7488,8}{7488,8+13,8} \approx 0,998$$

R

2

d) In this particular case it would be better to use correlation matrix, which implies that the data have been standardized.

The reason is that the variances of the sales and profit differ very much, which results in that the first PC catch too much of the variance and the second PC do not catch any of it at all.

It is not strange that sales catch that much of variance since it is at the top of the financial statement while profit is at the bottom, thus the values will differ significantly.

Therefore, it is best to use the correlation matrix when the variances differ a lot and covariance matrix when they do not differ a lot.

unless the differences represent relative importance of the variables

3

15

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 5

3a) 1. The means of indicators, common factors, unique factors are assumed to be zero.

$$E(x_j) = 0 \qquad E(F_j) = 0 \qquad E(\varepsilon_j) = 0$$

2. The variance of the indicators and common factors are assumed to be 1.

$$Var(x_j) = 1 \qquad Var(F_j) = 1$$

3. The unique factors are uncorrelated with themselves and with the indicators.

$$Cov(\varepsilon_j, \varepsilon_i) = 0 \qquad Cov(x_i, \varepsilon_i) = 0$$

3

b) The unique variances are equal to 1 − communalities.

Communalities $= \lambda_1^2 + \lambda_2^2$, since we do not have any correlation between the common factors. R

Also the reason why it is 1 − communalities is because the variance is equal to 1 by assumption. R

1. $1-(0,14^2 + 0,69^2) = 0,5043$
2. $1-((-0,64)^2 + (0,61)^2) = 0,283$
3. $1-(0,56^2 + 0,12^2) = 0,672$
4. $1-(0,8^2 + 0,16^2) = 0,3344$
5. $1-((-0,07)^2 + (-0,58)^2) = 0,6587$
6. $1-(0,49^2 + 0,54^2) = 0,4683$

R

4

c)

12	$0,14 \cdot (-0,64) + 0,69 \cdot 0,61$	$= 0,3313$
13	$0,14 \cdot 0,56 + 0,69 \cdot 0,12$	$= 0,1612$
14	$0,14 \cdot 0,8 + 0,69 \cdot 0,16$	$= 0,2224$
15	$0,14 \cdot (-0,07) + 0,69 \cdot (-0,58)$	$= -0,41$
16	$0,14 \cdot 0,49 + 0,69 \cdot 0,54$	$= 0,4412$
23	$-0,64 \cdot 0,56 + 0,61 \cdot 0,12$	$= -0,2852$
24	$-0,64 \cdot 0,8 + 0,61 \cdot 0,16$	$= -0,4144$
25	$-0,64 \cdot (-0,07) + 0,61 \cdot (-0,58)$	$= -0,309$
26	$-0,64 \cdot 0,49 + 0,61 \cdot 0,54$	$= 0,0158$
34	$0,56 \cdot 0,8 + 0,12 \cdot 0,16$	$= 0,4672$
35	$0,56 \cdot (-0,07) + 0,12 \cdot (-0,58)$	$= -0,1088$
36	$0,56 \cdot 0,49 + 0,12 \cdot 0,54$	$= 0,3392$
45	$0,8 \cdot (-0,07) + 0,16 \cdot (-0,58)$	$= -0,1488$
46	$0,8 \cdot 0,49 + 0,16 \cdot 0,54$	$= 0,4784$
56	$-0,07 \cdot 0,49 + (-0,58) \cdot 0,54$	$= -0,3475$

Estimated corr. matrix:

$$\tilde{R} = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 1 & & & & & \\ 0,3313 & 1 & & & & \\ 0,1612 & -0,2852 & 1 & & & \\ 0,2224 & -0,4144 & 0,4672 & 1 & & \\ -0,41 & -0,309 & -0,1088 & -0,1488 & 1 & \\ 0,4412 & 0,0158 & 0,3392 & 0,4782 & -0,3472 & 1 \end{bmatrix} \end{array}$$

R

5

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-HOX Sheet number: 6

2 d) First of all, if a varimax rotation was performed instead of a Quartimax rotation, then the focus would instead only be on the first factor with near zero pattern loadings on the second factor. Why?

This would result in different values for the unique variance and the reproduced correlation matrix. ✓

Second, it will depend on if the EFA was performed with a PCF or a PAF.

If it was performed by PCF, then the values will not change because you assume that the communalities are one and then perform a PCA. The PCs that are chosen are based on Horn's parallel analysis, scree plot, and Kaiser's rule, which becomes the factors and the PCs not chosen becomes the unique variances. They will never change.

But if a PAF is performed the values in b) and c) will change. Since PAF is an itterative PCF, where the found communalities are placed on the diagonal of the correlation matrix and then a new PCA is performed which result in different values. This will continue until the convergency criteria is met.

1

13

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 7

4a)

	1	2	3	4	5
1	0				
2	8,0623	0			
3	2	9,2195	0		
4	6,4031	3,1623	8,0623	0	
5	3,6056	4,4721	5	3,1623	0

The smallest number is 2, therefore will 1 and 3 become a cluster.

	13	2	4	5
13	0			
2	9,2195	0		
4	8,0623	3,1623	0	
5	5	4,4721	3,1623	0

$d_{(13)2} = \max(d_{12}, d_{23}) = \max(8{,}0623, 9{,}2195) = 9{,}2195$

$d_{(13)4} = \max(d_{14}, d_{34}) = \max(6{,}4031, 8{,}0623) = 8{,}0623$

$d_{(13)5} = \max(d_{15}, d_{35}) = \max(3{,}6056, 5) = 5$

R

The smallest value has both 2,4 and 4,5. Therefore 2,4 was chosen by randomly selecting one of them.

The new matrix will be

	13	24	5
13	0		
24	9,2195	0	
5	5	4,4721	0

$d_{(24)13} = \max(d_{2(13)}, d_{4(13)}) = \max(9{,}2195, 8{,}0623) = 9{,}2195$

$d_{(24)5} = \max(d_{25}, d_{45}) = \max(4{,}4721, 3{,}1623) = 4{,}4721$

R

The smallest value is 4,4721, therefore 24 and 5 will become a cluster.

The new matrix will be

R

	13	245
13	0	
245	9,2195	0

$d_{(245)13} = \max(d_{24(13)}, d_{5(13)}) = \max(9{,}2195, 5) = 9{,}2195$

R

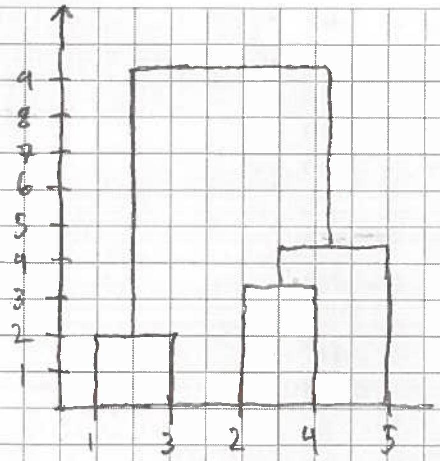
The complete linkage method is completed and the two clusters are (13) and (245).

7

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 8

4 b)



4

c) Cluster Analysis tries to group observation into groups were the observation within the group is homogenous and between groups they should differ as much they can.

So it is sort of an reduction technique, because we can say that a whole group of observation represent one thing and another group something else.

Another reduction technique is PCA, which aim to reduce p original variables to k principal components, which can explain a large amount of the original variables variation.

PCA do not work if the original variables are uncorrelated, which CA also needs, because then we cannot create groups since there is no homogeneity between the observations.

But PCA reduces the original variables to PCs which explain a large amount of variation, while CA does not.

They can complement each other by first performing a PCA and then use the PC scores to form clusters.

4

15

SU, DEPARTMENT OF STATISTICS

Room: UG1 Anonymous code: 0008-AOX Sheet number: 9

5a) Not any of the three variable seem to discriminate well jointly. All three graphs show no good seperation between the bankrupt and nonbankrupt.

Individually only $x_2 = NI/TA$ seem o seperate a little bit but not very good though, which can be seen in the first and third graph. (Bankrupt tend to have negative net income, ow x_3 and negative x_1 though)

3

b) $\gamma^T x = z$:

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}-0{,}45\\-0{,}41\\1{,}09\end{pmatrix} = -1{,}121411$$

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}0{,}51\\0{,}1\\2{,}49\end{pmatrix} = 3{,}002623$$

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}-0{,}56\\-0{,}31\\1{,}51\end{pmatrix} = -0{,}528879$$

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}0{,}08\\0{,}02\\2{,}01\end{pmatrix} = 1{,}825381$$

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}0{,}06\\0{,}02\\1{,}01\end{pmatrix} = 0{,}976325$$

$$(1{,}1278 \quad 3{,}6946 \quad 0{,}8265)\begin{pmatrix}0{,}38\\0{,}11\\3{,}27\end{pmatrix} = 3{,}537625$$

R 4

c)
$$Z \geq \frac{\bar{Z}_1 + \bar{Z}_2}{2} + \ln\left[\frac{P_2}{P_1} \cdot \frac{C(1|2)}{C(2|1)}\right]$$

$$Z \geq \frac{0{,}7809 + 2{,}6143}{2} + \ln\left[\frac{0{,}5}{0{,}5} \cdot \frac{1}{3}\right]$$

$$Z \geq 1{,}6826 + \ln\frac{1}{3}$$

$$Z \geq 0{,}584$$

1 = not bankrupt
0 = Bankrupt

	Predicted	Actual
Z_1	0	Bankrupt
Z_2	1	Non bankrupt
Z_3	0	Bankrupt
Z_4	1	Non bankrupt
Z_5	1	Bankrupt
Z_6	1	Non bankrupt

3

SU, DEPARTMENT OF STATISTICS

Room: UG Anonymous code: 0008-AOX Sheet number: 10

5d)

		Predicted	
		Non bankrupt	Bankrupt
Actual	Non bankrupt	3	0
	bankrupt	1	2

$$\text{Accuracy} = \frac{TP+TN}{TP+TN+FP+FN} = \frac{3+2}{3+0+1+2} = \frac{5}{6}$$

4

14